Mathematics

Lissajous figures

(Belgium)

Background

Lissajous figures (or Bowditch curves) are patterns produced by the intersection of two sinusoidal curves, the axes of which are at right angles to each other.

They were first studied by the American mathematician Nathaniel Bowditch in 1815 and then by French mathematician Jules-Antoine Lissajous in 1857–58.

Lissajous used a narrow stream of sand pouring from the base of a compound pendulum to produce the curves, as can be seen in Figures 1 and 2.

Mathematically, Lissajous Figures are a special case of parametric equations, where x and y are in the following form:

$$x = A \sin(at + \delta)$$
$$y = B \sin(bt + \gamma)$$

You will need:

- ✓ PVC pipes and connectors (figure 3) or clamp stands and a pipe
- ✓ bottle,
- ✓ sand or salt
- ✓ string
- ✓ black card,
- ✓ GeoGebra (app)

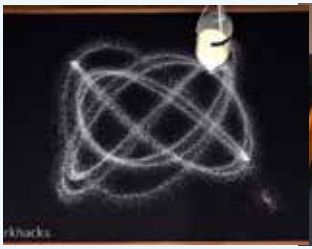

Figure 1

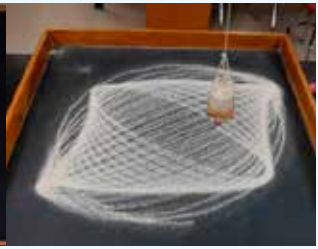

Figure 2

Follow these steps:

1. Connect the PVC pipes together as in Figure 3.
2. Make a holder for the sand by either using a cone made from card/plastic (Figure 3) or a bottle with a narrow hole in the lid (Figures 1 and 2).
3. Tie string in loops to the top of the stand which can be moved.
4. Attach another string in a loop to the first string, as in Figure 3, and then to the bottle itself.
5. Cover the hole with your finger and add fine sand or salt to the bottle.
6. Pull the bottle to a corner and release.
7. Repeat for different positions of the bottle or change string length/positions.

Figure 3

So what happened?

The sand traces out Lissajous patterns. If the frequency and phase angle of the two curves are identical, the resultant is a straight line. By varying the phase relation, ellipses are formed with varying angular positions. A phase difference of 90° (or 270°) produces a circle around the origin. If the curves are out of phase and differing in frequency, intricate meshing figures are formed.